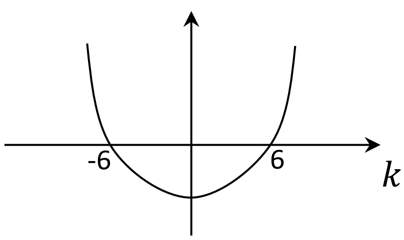
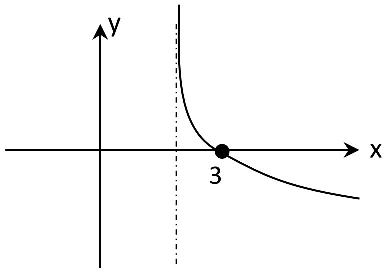
<u>CfE Higher Maths – December Assessment – Marking Scheme</u>

Q1	(a)			
		$\begin{array}{r\|rrrr} -1 & 6 & 7 & a & b \\ & & -6 & -1 & (1-a) \\ \hline & 6 & 1 & (a-1) & b+1-a \end{array}$	• Division by $(x+1)$ to achieve a remainder and equate to 0	1
		b + 1 – a = 0		
		$\begin{array}{r\|rrrr} 2 & 6 & 7 & a & b \\ & & 12 & 38 & 76+2a \\ \hline & 6 & 19 & (38+a) & 76+2a+b \end{array}$	• Division by $(x-2)$ to achieve a remainder and equate remainder to 72	1
		$76+2a+b = 72$		
		$\begin{aligned} 2a+b &= -4 \\ a-b &= 1 \end{aligned}$	• Know to use the application of the above to solve two equations in $a \text{ and } b$.	1
		$\begin{aligned} 3a &= -3 \\ a &= -1 \\ b &= -2 \end{aligned}$	• Solve the two equations correctly to get $a=-1 \quad b=-2$	1
			(Note: Can use Long Division)	
	(b)	$6x^3 + 7x^2 + ax + b = 0$		
		<=> $(x+1)(6x^2+x-2) = 0$	• Use a division outcome to achieve a • partially factorised polynomial in x and equate to 0	2
		<=> $(x+1)(3x+2)(2x-1) = 0$	• Fully factorise the polynomial correctly	1
		<=> $x = -1,\ 3x = -2,\ 2x-1 = 0$		
		$x = -1 \quad x = \frac{-2}{3} \quad x = \frac{1}{2}$	• Solve correctly for x (3 solutions)	1

Q2	$f(x) = \frac{2}{3}\sin(3x-1)$ $f'(x) = \frac{2}{3}\cos(3x-1) \times 3$ $f'(x) = 2\cos(3x-1)$	• Correctly apply the chain rule • Correctly simplify	1 1
Q3	$g(x) = x(x-3)^2$ $= x(x^2-6x+9)$ $= x^3 - 6x^2 + 9x$ $g'(x) = 3x^2 - 12x + 9$ Stationary points when $g'(x) = 0$ $3x^2 - 12x + 9 = 0$ $3(x^2 - 4x + 3) = 0$ $3(x-3)(x-1) = 0$ $x = 3, \quad x = 1$ Stationary points at (3, 0) and (1, 4)	• Finds $g'(x)$ • sets derivative to zero • Solves for x • Finds corresponding y-values	1 1 1 1
Q4	$x^2 - kx + 9 = 0$ Discriminant is $\left(b^2 - 4ac\right)$ $a = 1, \; b = -k, \; c = 9$ $b^2 - 4ac = k^2 - 36$ No Real roots: $k^2 - 36 < 0$ $(k+6)(k-6) < 0$  No Real roots when $-6 < k < 6$	• Arrange equation into quadratic format, set to zero. • Express discriminant in terms of k $(k^2 - 36)$ • State that for solution $(k^2 - 36) < 0$ • Solve inequality correctly $-6 < k < 6$	1 1 1 1

Q5	$5cosx = 6cos2x + 4$ $= 12cos^2x - 6 + 4$ <=> $12cos^2x - 5cosx - 2 = 0$ $(4cosx + 1)(3cosx - 2) = 0$ $4cosx + 1 = 0$, $cosx = -\frac{1}{4}$ | $3cosx - 2 = 0$, $cosx = \frac{2}{3}$ $0 \le x \le \pi$ so answers From 1st and 2nd quadrants only $x = 1.82$ | $x = 0.841$	• Use the double angle angle formula correctly to expand $6cos2x$ • Form a quadratic equation, in x (set to 0) • Factorise correctly • Identify two values for $cosx$ Note: if assumed $0 \le x \le 2\pi$ with 2 extra solutions, lose last mark • Find the 1st and 2nd quadrant solutions in radians correctly.	1 1 1 1 1
Q6	(a)  	Sketch the reflected curve clearly showing • The  shape • The point (3,0) or 3, marked on the x-axis • The correct-"ish" curvature – i.e. asymptotic – "ish" to the line x =2 (which does not need to be drawn)	1 1 1

Q7	$f(x) = \frac{x^3-2\sqrt{x}}{x}, x > 0$ $f(x) = \frac{x^3}{x} - \frac{2x^{\frac{1}{2}}}{x}$ $= x^2 - 2x^{-1/2}$ $f'^{(x)} = 2x + x^{-3/2}$ $= 2x + \frac{1}{x^{3/2}}$ Gradient of tangent $f'\left(\frac{1}{9}\right) = 2\left(\frac{1}{9}\right) + \frac{1}{\left(\frac{1}{9}\right)^{3/2}}$ $= \frac{2}{9} + \left(\frac{9}{1}\right)^{3/2}$ $= \frac{2}{9} + 3^3 \quad = 27\frac{2}{9}$	• Rearrange expression into differentiable form • Differentiate 1st term correctly • Differentiate 2nd term correctly • Rearrange to have positive indices • Substitute 1/9 into equation correctly • Evaluate $f'\left(\frac{1}{9}\right)$ correctly as an exact value	1 2 1 1 1
Q8	$p \cdot (q + r - p)$ $= p \cdot q + p \cdot r - p \cdot p$ $= \lvert p\rvert\lvert q\rvert cos60 + \lvert p\rvert\lvert r\rvert cos60 - \lvert p\rvert\lvert p\rvert cos0$ $= \left(3 \times 3 \times \frac{1}{2}\right) + \left(3 \times 3 \times \frac{1}{2}\right) - (3 \times 3 \times 1)$ $= \frac{9}{2} + \frac{9}{2} - 9$ $= 0$	• Able to expand brackets as dot product sums • know that dot product is evaluated using $\lvert a\rvert\lvert b\rvert cos\theta$ • know that angles between $(p\ and\ q)$ and $(p\ and\ r)$ are both 60^0 and angle between $(p\ and\ p)$ is 0^0 • evaluate expression correctly	1 1 1 1